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Now if the functions Ω and W be introduced as new variables in place of x and y, and put

$$\bar{x}=\Omega(x,\,y),\qquad \bar{y}=W(x,\,y),$$

$$\bar{x}_1=\Omega(x_1,\,y_1),\qquad \bar{y}_1=W(x_1,\,y_1),$$

the group takes the simple form

$$\bar{x}_1=\bar{x},\qquad \bar{y}_1=\bar{y}+t\,; \tag{25}$$

that is, the form of a group of translations.

The new variables $\bar{x}$ and $\bar{y}$ are called the *canonical variables* of the one parameter group (24), and (25) is called the *canonical form* of (24).

The reader will find many interesting details relative to the points and theorems here discussed, in the third chapter of LIE's lectures on differential equations.

Princeton University, 21 February, 1898.

[To be Continued.]

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

84. Proposed by SYLVESTER ROBINS, North Branch Depot, N. J.

Show how to find sides, integral, fractional, and irrational for twenty-four triangles, each one containing 330 square yards.

Solution by the PROPOSER.

1. $\Delta^2=330^2=2\times2\times3\times3\times5\times5\times11\times11=(5\times11)(2\times3\times5)(2\times11)\times3$ $=55\times30.22\times3$. ∴ Sides of triangle are 25, 33, and 52.

2. $\Delta^2=330^2=(2\times5\times11)(3\times3\times11)(2\times5)=110\times99\times10\times1$. ∴ Sides of Δ are 11, 100, and 109.

3. $\Delta^2=330^2=(2\times5\times5)(3\times11)(11)(2\times3)=50\times33\times11\times6$. ∴ Sides of Δ are 17, 39, 44.

4. $330^2=(2\times3\times11)(5\times11)(2\times3)(5)=66\times55\times6\times5$. ∴ Sides of Δ are 11, 60, and 61.

5. $330^2=(2\times2\times2\times2\times3)\left(\frac{5\times11}{2}\right)(3\times5)(\frac{11}{2})=48\times27\frac{1}{2}\times15\times5\frac{1}{2}$. ∴ Sides of Δ are $20\frac{1}{2}$, 33, and $42\frac{1}{2}$.

6. $330^2=(2\times2\times11)(2\times2\times5)\left(\frac{3\times11}{2}\right)\left(\frac{3\times5}{2}\right)=44\times20\times16\frac{1}{2}\times7\frac{1}{2}$. $\therefore$ Sides of $\triangle$ are 24, $27\frac{1}{2}$, and $36\frac{1}{2}$.

7. $330^2=(2\times2\times3\times5)\left(\frac{3\times5\times11}{2\times2}\right)(2\times2\times2\times2)\left(\frac{11}{2\times2}\right)=60\times41\frac{1}{4}\times16\times2\frac{3}{4}$. $\therefore$ Sides of $\triangle$ are $18\frac{3}{4}$, 44, and $57\frac{1}{4}$.

8. $330^2=(3\times3\times11)(2\times3\times3\times5)\left(\frac{2\times11}{3}\right)(\frac{5}{3})=99\times90\times7\frac{1}{3}\times1\frac{2}{3}$. $\therefore$ Sides of $\triangle$ are 9, $91\frac{2}{3}$, and $97\frac{1}{3}$.

9. $\triangle^2=165^2=3\times3\times5\times5\times11\times11=(3\times11)(2\times2\times5)\left(\frac{3\times5}{2}\right)(\frac{11}{2})=33\times20\times7\frac{1}{2}\times5\frac{1}{2}$. $\therefore$ Sides of $\triangle=165$ are 13, $25\frac{1}{2}$, and $27\frac{1}{2}$, and sides of $\triangle=330$ are $13\sqrt{2}$, $25\frac{1}{2}\sqrt{2}$, and $27\frac{1}{2}\sqrt{2}$.

10. $165^2=(2\times3\times5)\left(\frac{5\times11}{2\times2}\right)\left(\frac{3\times11}{2\times2}\right)(2\times2\times2)=30\times13\frac{3}{4}\times8\frac{1}{4}\times8$. $\therefore$ Sides of $\triangle$ are $16\frac{1}{4}$, $21\frac{3}{4}$, and 22. Sides of similar $\triangle$ containing 330 must be $16\frac{1}{4}\sqrt{2}$, $21\frac{3}{4}\sqrt{2}$, and $22\sqrt{2}$.

11. $\triangle^2=110^2=2\times2\times5\times5\times11\times11=(2\times2\times2\times3)\left(\frac{5\times11}{2\times3}\right)\left(\frac{3\times5}{2}\right)\left(\frac{2\times11}{3}\right)=24\times9\frac{1}{6}\times7\frac{1}{2}\times7\frac{1}{3}$. $\therefore$ Sides of $\triangle=110$ are $14\frac{5}{6}$, $16\frac{1}{2}$, and $16\frac{2}{3}$. Sides of $\triangle=330$ are $14\frac{5}{6}\sqrt{3}$, $16\frac{1}{2}\sqrt{3}$, and $16\frac{2}{3}\sqrt{3}$.

12. $110^2=\left(\frac{2\times5\times11}{3}\right)(2\times3\times5)(\frac{11}{3})(3)=36\frac{2}{3}\times30\times3\frac{2}{3}\times3$. $\therefore$ Sides are $6\frac{2}{3}$, 33, and $33\frac{2}{3}$, and sides of similar $\triangle=330$ must be $6\frac{2}{3}\sqrt{3}$, $33\sqrt{3}$, and $33\frac{2}{3}\sqrt{3}$.

13. $\triangle^2=66^2=2\times2\times3\times3\times11\times11=(2\times11)(11)(3\times3)(2)=22\times11\times9\times2$. $\therefore$ Sides of $\triangle=66$ are 11, 13, and 20, and sides of $\triangle=330$ are $11\sqrt{5}$, $13\sqrt{5}$, and $20\sqrt{5}$.

14. $66^2=(2\times3\times3\times3)(3\times3\times3)\left(\frac{2\times11\times11}{3\times3}\right)\left(\frac{1}{3\times3}\right)=54\times27\times26\frac{8}{9}\times\frac{1}{9}$. $\therefore$ Sides are 27, $27\frac{1}{9}$, and $53\frac{8}{9}$, and sides of $\triangle=330$ are $27\sqrt{5}$, $27\frac{1}{9}\sqrt{5}$, and $53\frac{8}{9}\sqrt{5}$.

15. $66^2=(3\times11)\left(\frac{3\times11}{2}\right)(2\times2\times2\times2)(\frac{1}{2})=33\times16\frac{1}{2}\times16\times\frac{1}{2}$. $\therefore$ Sides of $\triangle=66$ are $16\frac{1}{2}$, 17, and $32\frac{1}{2}$. Necessarily, sides of similar $\triangle=330$ are $16\frac{1}{2}\sqrt{5}$, $17\sqrt{5}$, and $32\frac{1}{2}\sqrt{5}$.

16. $66^2=(2\times2\times2\times3)\left(\frac{3\times11}{2}\right)(\frac{11}{2})(2)=24\times16\frac{1}{2}\times5\frac{1}{2}\times2$. $\therefore$ Sides are $7\frac{1}{2}$, $18\frac{1}{2}$, and 22, and sides of similar $\triangle=330$ are $7\frac{1}{2}\sqrt{5}$, $18\frac{1}{2}\sqrt{5}$, $22\sqrt{5}$.

17. $\triangle^2=55^2=(5\times5\times11\times11)=(2\times3\times3)\left(\frac{5\times11}{2\times3}\right)(\frac{11}{2})\left(\frac{2\times5}{3}\right)=18\times9\frac{1}{6}\times5\frac{1}{2}\times3\frac{1}{3}$. $\therefore$ Sides are $8\frac{5}{6}$, $12\frac{1}{2}$, and $14\frac{2}{3}$, and sides of $\triangle=330$ are $8\frac{5}{6}\sqrt{6}$, $12\frac{1}{2}\sqrt{6}$, and $14\frac{2}{3}\sqrt{6}$.

18. $55^2=(2\times2\times5)\left(\frac{5\times11}{2\times3}\right)(3\times3)\left(\frac{11}{2\times3}\right)=20\times9\frac{1}{6}\times9\times1\frac{5}{6}$. $\therefore$ Sides of $\triangle=55$ are $10\frac{5}{6}$, 11, and $18\frac{1}{6}$, and sides of $\triangle=330$ are $10\frac{5}{6}\sqrt{6}$, $11\sqrt{6}$, $18\frac{1}{6}\sqrt{6}$.

19. $\triangle^2=(47\frac{1}{7})^2=(\frac{330}{7})^2=(3\times11)(2\times3\times5)(\frac{11}{7})\left(\frac{2\times5}{7}\right)=33\times30\times1\frac{4}{7}\times1\frac{3}{7}$. $\therefore$ Sides are 3, $31\frac{3}{7}$, and $31\frac{4}{7}$, and sides of $\triangle=330$ are $3\sqrt{7}$, $31\frac{3}{7}\sqrt{7}$, and $31\frac{4}{7}\sqrt{7}$.

20. $\triangle^2=(47\frac{1}{7})^2=(\frac{330}{7})=\left(\frac{2\times5\times11}{7}\right)\left(\frac{3\times11}{7}\right)(2\times3)(5)=15\frac{5}{7}\times4\frac{5}{7}\times6\times5$. $\therefore$ Sides are 11, $9\frac{5}{7}$, and $10\frac{5}{7}$, and sides of $\triangle=330$ are $11\sqrt{7}$, $9\frac{5}{7}\sqrt{7}$, and $10\frac{5}{7}\sqrt{7}$.

21. $(47\frac{1}{7})^2=(\frac{330}{7})^2=(2\times11)\left(\frac{2\times2\times2\times3\times5}{7}\right)(\frac{5}{2})\left(\frac{3\times11}{2\times7}\right)=22\times17\frac{1}{7}\times2\frac{1}{2}\times2\frac{5}{14}$. $\therefore$ Sides are $4\frac{6}{7}$, $19\frac{1}{2}$, and $19\frac{9}{14}$, and sides of $\triangle=330$ are $4\frac{6}{7}\sqrt{7}$, $19\frac{1}{2}\sqrt{7}$, $19\frac{9}{14}\sqrt{7}$.

22. $(47\frac{1}{7})^2=(\frac{330}{7})^2=\left(\frac{3\times11}{2}\right)\left(\frac{2\times2\times3\times5}{7}\right)(2\times2)\left(\frac{5\times11}{2\times7}\right)=16\frac{1}{2}\times8\frac{4}{7}\times4\times3\frac{13}{14}$. $\therefore$ Sides are $7\frac{13}{14}$. $12\frac{1}{2}$, and $12\frac{4}{7}$, and sides of $\triangle=330$ are $7\frac{13}{14}\sqrt{7}$, $12\frac{1}{2}\sqrt{7}$, and $12\frac{4}{7}\sqrt{7}$.

23. $\triangle^2=33^2=(3\times3\times11\times11)=\left(\frac{3\times11}{2}\right)(11)(2\times2)(\frac{3}{2})=16\frac{1}{2}\times11\times4\times1\frac{1}{2}$. $\therefore$ Sides are $5\frac{1}{2}$, $12\frac{1}{2}$, and 15, and sides of similar $\triangle=330$ are $5\frac{1}{2}\sqrt{10}$, $12\frac{1}{2}\sqrt{10}$, and $15\sqrt{10}$.

24. $33^2=\left(\frac{3\times3\times5}{2}\right)\left(\frac{11\times11}{2\times5}\right)(2\times5)(\frac{2}{5})=22\frac{1}{2}\times12\frac{1}{10}\times10\times\frac{2}{5}$. $\therefore$ Sides are $10\frac{2}{5}$, $12\frac{1}{2}$, and $22\frac{1}{10}$, and sides of $\triangle=330$ are $10\frac{2}{5}\sqrt{10}$, $12\frac{1}{2}\sqrt{10}$, and $22\frac{1}{10}\sqrt{10}$.

25. $\triangle^2=30^2=2\times2\times3\times3\times5\times5=(3\times5)(2\times5)(3)(2)=15\times10\times3\times2$. $\therefore$ Sides of $\triangle=30$ are 5, 12, and 13, and sides of $\triangle=330$ are $5\sqrt{11}$, $12\sqrt{11}$, and $13\sqrt{11}$.

26. $30^2=(2\times2\times5)(2\times2\times3)\left(\frac{3\times5}{2}\right)(\frac{1}{2})=20\times12\times7\frac{1}{2}\times\frac{1}{2}$. $\therefore$ Sides are 8, $12\frac{1}{2}$, and $19\frac{1}{2}$, and sides of $\triangle=330$ are $8\sqrt{11}$, $12\frac{1}{2}\sqrt{11}$, and $19\frac{1}{2}\sqrt{11}$.

27. $30^2=(2\times2\times2\times2\times2)(2\times3\times5)\left(\frac{5}{2\times2}\right)\left(\frac{3}{2\times2}\right)=32\times30\times1\frac{1}{4}\times\frac{3}{4}$. $\therefore$ Sides of $\triangle=30$ are 2, $30\frac{3}{4}$, and $31\frac{1}{4}$, and sides of $\triangle=330$ are $2\sqrt{11}$, $30\frac{3}{4}\sqrt{11}$, and $31\frac{1}{4}\sqrt{11}$.

28. $\triangle^2=(25\frac{5}{13})^2=(\frac{330}{13})^2=(3\times5)(11)\left(\frac{2\times3\times5}{13}\right)\left(\frac{2\times11}{13}\right)=15\times11\times2\frac{4}{13}\times1\frac{9}{13}$. $\therefore$ Sides are 4, $12\frac{9}{13}$, and $13\frac{4}{13}$, and sides of $\triangle=330$ are $4\sqrt{13}$, $12\frac{9}{13}\sqrt{13}$, and $13\frac{4}{13}\sqrt{13}$.

29. $(\frac{330}{13})^2=\left(\frac{2\times3\times5\times11}{13}\right)(2\times11)(3)(\frac{5}{13})=25\frac{5}{13}\times22\times3\times\frac{5}{13}$. $\therefore$ Sides are $3\frac{5}{13}$, $22\frac{5}{13}$, and 25, and sides of $\triangle=330$ are $3\frac{5}{13}\sqrt{13}$, $22\frac{5}{13}\sqrt{13}$, and $25\sqrt{13}$.

30. $\triangle^2=(23\frac{4}{7})^2=(\frac{165}{7})^2=\frac{3\times3\times5\times5\times11\times11}{7\times7}=\left(\frac{3\times5\times11}{2\times7}\right)(\frac{11}{2})\left(\frac{2\times2\times3\times5}{2\times7}\right)(2)=11\frac{11}{14}\times5\frac{1}{2}\times4\frac{4}{14}\times2$. $\therefore$ Sides of $\triangle=23\frac{4}{7}$ are $6\frac{2}{7}$, $7\frac{1}{2}$, and $9\frac{11}{14}$, and sides of $\triangle=330$ are $6\frac{2}{7}\sqrt{14}$, $7\frac{1}{2}\sqrt{14}$, and $9\frac{11}{14}\sqrt{14}$.

31. $(23\frac{4}{7})^2=(\frac{165}{7})^2=(2\times11)\left(\frac{3\times5\times11}{2\times7}\right)(2\times5)\left(\frac{3}{2\times7}\right)=22\times\frac{11}{14}\times10\times\frac{3}{14}$.
$\therefore$ Sides are $10\frac{3}{14}$, 12. and $21\frac{1}{14}$, and sides of $\triangle=330$ are $10\frac{3}{14}\sqrt{14}$, $12\sqrt{14}$, and $21\frac{1}{14}\sqrt{14}$.

32. $\triangle^2=22^2=2\times2\times11\times11=(2\times11)(2\times3\times3)(\frac{11}{3})(\frac{1}{3})=22\times18\times3\frac{2}{3}\times\frac{1}{3}$. $\therefore$ Sides are 4, $18\frac{1}{3}$. and $21\frac{2}{3}$, and sides of $\triangle=330$ are $4\sqrt{15}$, $18\frac{1}{3}\sqrt{15}$, and $21\frac{2}{3}\sqrt{15}$.

33. $\triangle^2=15^2=3\times3\times5\times5=(2\times5)(2\times3)(\frac{5}{2})(\frac{3}{2})=10\times6\times2\frac{1}{2}\times1\frac{1}{2}$. $\therefore$ Sides are 4, $7\frac{1}{2}$, and $8\frac{1}{2}$, and, of course, sides of $\triangle=330$ are $4\sqrt{22}$, $7\frac{1}{2}\sqrt{22}$, and $8\frac{1}{2}\sqrt{22}$.

34. $\triangle^2=10^2=2\times2\times5\times5=(3\times3)\,(5)\left(\frac{2\times5}{3}\right)(\frac{2}{3})=9\times5\times3\frac{1}{3}\times\frac{2}{3}$. $\therefore$ Sides are 4, $5\frac{2}{3}$, and $8\frac{1}{3}$, and sides of $\triangle=330$ are $4\sqrt{33}$, $5\frac{2}{3}\sqrt{33}$, and $8\frac{1}{3}\sqrt{33}$.

ALGEBRA.

79. Proposed by C. W. M. BLACK, A. M., Professor of Mathematics, Wesleyan Academy, Wilbraham, Mass.

Of n persons A, B, C, etc., A gives to the others as much as each of them already has; then B gives to the others as much as each then has; and so on for each in turn. Finally, A, B, C, etc., have respectively a, b, c, etc., dollars. How much had each at first?

I. Solution by the PROPOSER.

Let $a+b+c+$etc......$=s$, which is constant. Consider the rth person, K, who has at the end k dollars. Let x=number of dollars he had at first. Before he gives away any, his original amount is doubled $r-1$ times and comes to equal $2^{r-1}x$. At that time all the rest must together have $s-2^{r-1}x$, and if he gives away that amount he has left $2^{r-1}x-(s-2^{r-1}x)=2^rx-s$. Afterwards his money is doubled $n-r$ times and finally equals $2^{n-r}(2^rx-s)=k$.

$\therefore x=(k+2^{n-r}s)/2^n$.

II. Solution by I. H. BRYANT, A. M., Instructor in Mathematics in Fort Smith High School, Fort Smith, Ark.; G. B. M. ZERR, A. M., Ph. D., Russell College, Lebanon, Va.; and J. SCHEFFER, A. M., Hagerstown, Md.

Let x_1, x_2, x_3, x_4, ..., x_n be the respective amounts each had at first.

Let $s=x_1+x_2+x_3+......+x_n=a+b+c+......(1)$.

$\therefore 2^{n-1}(2x_1-s)=a$ .....(2).

$2^{n-2}(4x_2-s)=b$ ......(3).

$2^{n-3}(8x_3-s)=c$ ......(4).

....

$(2^nx_n-s=n$.

(2), (3), (4), (5), etc., in (1) gives, $x_1=(a+2^{n-1}s)/2^n$, $x_2=(b+2^{n-2}s)/2^n$, $x_3=(c+2^{n-3}s)/2^n$, $x_4=(d+2^{n-4}s)/2^n$, ...$x_n=(n+s)/2^n$.

III. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x_1, x_2, x_3, ... x_{n-1}, x_n=the money that A, B, C, etc., respectively, had at first. Put $x_n=r$, $x_{n-1}=2r-s$, $x_{n-2}=2x_{n-1}-s=4r-3s$, $x_{n-3}=2x_{n-2}-s=8r-7s$, etc., etc. The law that governs the forming of these values makes $x_1=2^{n-1}(r-s)+s$; $x_2=2^{n-2}(r-s)+s$; $x_3=2^{n-3}(r-s)+s$; $x_4=2^{n-4}(r-s)+s$;